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A Polynomial First Order Action for the Dirichlet 3-brane

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Abstract

A new first order action for *type IIB* Dirichlet 3-brane is proposed. Its form is inspired by the superfield equations of motion obtained recently from the generalized action principle. The action involves auxiliary symmetric spin tensor fields. It seems promising for a reformulation of the generalized action in a structure most adequate for investigating the extrinsic geometry of the super-3-brane, but also for further studies of string dualities.

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1 Introduction

Dirichlet (super)-p-branes [1]–[12] recently have received much attention as objects related to nonperturbative superstring (or M-)theory physics, (see e.g. [2], [11]).

The original D-brane action is of Dirac–Born–Infeld (DBI) type [1, 4]

$$I_{DBI} = - \int_{\mathcal{M}_0} d^{p+1} \xi \sqrt{-\det(g_{mn} + \mathcal{F}_{mn})}, \quad (1)$$

where

$$g_{mn} = \partial_m X^{\underline{m}} \eta_{\underline{m}\underline{n}} \partial_n X^{\underline{n}}$$

is the induced metric, and the field \mathcal{F}_{mn} are the components of the 2-form field strength

$$\mathcal{F} = dA = d\xi^m \wedge d\xi^n \mathcal{F}_{nm}, \quad A = d\xi^m A_m. \quad (2)$$

For simplicity we will consider D-brane actions in a flat background here, though their curved space generalizations are straightforward.

The supersymmetric generalization of this action was found recently [5]–[8] and was used for constructing the generalized action [13] for Dirichlet superbranes [9], which allows to obtain the superfield equations of motion for super-D-p-branes [9]. The linearized form of such equations was discovered previously [3] in the frame of the so-called supersymmetric geometric approach [14, 15].

Within the last months several authors [10, 16, 17] for different reasons addressed the problem how to avoid the appearance of the square root in (1) by a transition to another action which becomes an analog of the so-called Polyakov action [18]. Such an action, found in fact for the first time in Ref. [19], offers the possibility to study rigid symmetries, duality properties and the strong coupling limit of D-branes. However, the action [19], used in [10, 16, 17], is still not of polynomial type. An auxiliary world volume tensor field and its inverse are necessary to write down the action.

The bosonic limit of the generalized action for the Dirichlet super-p-branes (Dp-branes) [9] is already of first order in the field strength of the world volume gauge field.

However, its structure is not completely identical [9] to the one for superstrings and *type I* superbranes [13, 15]. If we consider the pure bosonic limit (the so-called moving frame or Lorentz harmonic formulation of the bosonic branes, see [20, 14, 21]) for the latter in a flat background, it is of first order in all the dynamical fields. The bosonic limit of the generalized action of the Dirichlet superbranes, being of the first order in the auxiliary gauge field strength \mathcal{F} , remains of $p + 1$ -th order in the derivative of the embedding coordinate functions $X^{\underline{m}}(\xi^m)$ ($\underline{m} = 0, \dots, 9$; $m = 0, \dots, p$).

Thus the natural question to ask is whether it is possible to write the generalized action for super-D-p-branes and, hence, the moving frame action for D-branes in first order form.

The purpose of this note is to give an affirmative answer. We present this new first order form of the action for a particular, but especially interesting, case of a *type IIB* D3-brane. The basical requirement for such an action has been to reproduce the relation between induced and

intrinsic vielbeins, most adequate for the description of the embedding of superspaces [22, 9]. It is gratifying that this in a straightforward way leads even to a polynomial form of the action.

Our result provides a basis for an improved reformulation of the generalized action which, in turn, enables further studies of the extrinsic geometry of D-brane world volume superspace. It also seems to be a convenient new element for the investigation of duality problems.

In addition our result can be regarded as a preliminary step for obtaining a generalized action for the M -theory super-5-brane which is to be still beset by some problems [23].

In section 2 we summarize the information about the bosonic limit of the generalized action for $D = 10$ D-p-branes. The adequate world volume vielbein of section 3 is the basis for the construction of our new action in terms of adequate auxiliary fields in Section 4. In the Conclusion we also return to problems of the M-theory 5-brane.

2 Moving frame action and embedding equations for $D = 10$ Dirichlet p-branes.

The moving frame action for the bosonic D_p -brane [9] in flat $D = 10$ space-time reads ¹

$$S_D = \int_{\mathcal{M}^{p+1}} (\mathcal{L}_{p+1}^0 + \mathcal{L}_{p+1}^1) \quad (3)$$

$$\mathcal{L}_{p+1}^0 = \frac{1}{(p+1)!} E^{a_0} \wedge \dots \wedge E^{a_p} \epsilon_{a_0 \dots a_p} \sqrt{-\det(\eta_{ab} + F_{ab})} \quad (4)$$

$$\mathcal{L}_{p+1}^1 = Q_{p-1} \wedge \left[dA - \frac{1}{2} E^b \wedge E^a F_{ab} \right]. \quad (5)$$

In (4) Q_{p-1} is the Lagrange Multiplier $(p-1)$ -form,

$$E^a = dX^{\underline{m}}(\xi) u_{\underline{m}}^a(\xi) \quad (6)$$

is the pull-back of the $(p-1)$ components of a target space vielbein form ($\underline{m} = 0, \dots, 9$; $\underline{a} = 0, \dots, 9$; $a = 0, \dots, p$; $i = 1, \dots, 9-p$)

$$E^{\underline{a}} = dX^{\underline{m}} u_{\underline{m}}^{\underline{a}} = (E^a, E^i) \quad (7)$$

which is related to the holonomic vielbein $dX^{\underline{m}}$ by a Lorentz rotation parametrized by the matrix

$$(u_{\underline{m}}^{\underline{a}}) = (u_{\underline{m}}^a, u_{\underline{m}}^i) \in SO(1, 9) \Leftrightarrow u_{\underline{m}}^{\underline{a}} \eta^{\underline{m}\underline{n}} u_{\underline{n}}^{\underline{b}} = \eta^{\underline{a}\underline{b}} \quad (8)$$

This Lorentz rotation is chosen to adapt the vielbein (7) to the world volume of the (Dirichlet) p -brane in the sense that only $(p+1)$ components E^a of (7) enter into the action. Thus the $u_{\underline{m}}^{\underline{a}}$ depend on the world volume coordinates ξ^m and shall be regarded as (auxiliary) field variables (Lorentz harmonic variables, see [24, 25] and refs. in [14]). They are to be varied to obtain the equations of motion on equal footing with $X^{\underline{m}}$, taking into account the conditions (8). To avoid the introduction of the constraints (8) with Lagrange multipliers into the action, we can restrict

¹In this section for the simplicity we drop the dependence on all background fields, including the 2-form B_2 and the dilaton ϕ .

the variations of $u_{\underline{m}}^a$ to the space isomorphic to the Lie algebra of the Lorentz group [20]. Such variations shall be just infinitesimal Lorentz rotations

$$\delta u_{\underline{m}}^a = u_{\underline{m}}^b \mathcal{O}_{\underline{b}}^a, \quad \mathcal{O}^{ba} = -\mathcal{O}^{ab}. \quad (9)$$

This defines "admissible" derivatives of the harmonic variables

$$du_{\underline{m}}^a = u_{\underline{m}}^b \Omega_{\underline{b}}^a \quad (10)$$

as well, where

$$\Omega^{ab} = -\Omega^{ba} = \begin{pmatrix} \Omega^{ab} & \Omega^{aj} \\ -\Omega^{bi} & \Omega^{ij} \end{pmatrix} = u_{\underline{m}}^a du_{\underline{m}}^{bm} \quad (11)$$

is an $so(1, D-1)$ valued Cartan 1-form ².

Clearly, the definition (11) of the Cartan forms can be regarded as the statement that they are trivial $SO(1, 9)$ connections. The condition of vanishing for the corresponding $SO(1, 9)$ curvature (equivalent to (11)) coincides with the Maurer–Cartan equations

$$d\Omega^{ab} - \Omega_{\underline{c}}^a \wedge \Omega^{cb} = 0, \quad \Leftrightarrow \quad \begin{cases} \mathcal{D}\Omega^{ai} \equiv d\Omega^{ai} - \Omega_b^a \wedge \Omega^{bi} + \Omega^{aj} \wedge \Omega^{ji} = 0, \\ R^{ab} = d\Omega^{ab} - \Omega_c^a \wedge \Omega^{cb} = \Omega^{ai} \wedge \Omega^{bi}, \\ R^{ij} = d\Omega^{ij} + \Omega^{ij'} \wedge \Omega^{j'j} = -\Omega^{ai} \wedge \Omega_a^j, \end{cases} \quad (12)$$

Eqs. (12) for the forms $\Omega^{ai}, \Omega^{ab}, \Omega^{ij}$ (11) pulled back to the world volume give rise to the Peterson–Codazzi, Gauss and Ricci equations of surface theory [26].

Splitting the expression for the admissible variations in an $SO(1, p) \times SO(9-p)$ invariant way (cf. (7) (8), (11)), we get in particular

$$\delta u_{\underline{m}}^a = u_{\underline{m}}^b \mathcal{O}_b^a - u_{\underline{m}}^i \mathcal{O}^{ia} = u_{\underline{m}}^b i_\delta \Omega_b^a + u_{\underline{m}}^i i_\delta \Omega^{ai}. \quad (13)$$

Eq. (13) can be regarded as the contraction of the expression for admissible derivatives

$$du_{\underline{m}}^a = u_{\underline{m}}^b \Omega_b^a + u_{\underline{m}}^i \Omega^{ai}, \quad \Leftrightarrow \quad Du_{\underline{m}}^a \equiv du_{\underline{m}}^a - u_{\underline{m}}^b \Omega_b^a = u_{\underline{m}}^i i_\delta \Omega^{ai}. \quad (14)$$

For completeness let us present the analogous expression for the derivatives of the harmonic $u_{\underline{m}}^i$:

$$du_{\underline{m}}^i = -u_{\underline{m}}^j \Omega^{ji} + u_{\underline{m}a} \Omega^{ai} \quad \Leftrightarrow \quad Du_{\underline{m}}^i \equiv du_{\underline{m}}^i + u_{\underline{m}}^j \Omega^{ji} = u_{\underline{m}a} \Omega^{ai} \quad (15)$$

It turns out [9] that the parameter $\mathcal{O}^{ba} = i_\delta \Omega_{ba}$ (13) of the $SO(1, p)$ subgroup of the $SO(1, 9)$ can be identified with the world volume local Lorentz symmetry of the D-p-brane model and yields no independent equations of motion (Noether identity), while the variation $\mathcal{O}^{ai} = i_\delta \Omega_{ai}$ provides us with the equation (see [14, 13, 21, 9])

$$E^i \equiv dX^m u_{\underline{m}}^i = 0. \quad (16)$$

In Eq. (16) the statement that the vielbein E^a is adapted to the embedding is thus realized in a concrete way.

²The parameters of admissible variations \mathcal{O} can be considered as contractions of the Cartan forms (11) with the variation symbol δ , $\mathcal{O}^{ab} = i_\delta \Omega^{ab}$.

The additional assumption that the pull-backs of the remaining vielbein forms are linearly independent (which is a nondegeneracy condition on the embedding) means that, in general,

$$E^a \equiv dX^{\underline{m}} u_{\underline{m}}^a = e^b m_b^a, \quad (17)$$

where the matrix m_b^a is supposed to be nondegenerate $\det(m_b^a) \neq 0$. For any choice of the matrix m , for the induced metric

$$g_{mn} \equiv \partial_m X^{\underline{m}} \eta_{\underline{m}\underline{n}} \partial_n X^{\underline{m}} \equiv E_m^a \eta_{ab} E_n^b = E_m^a \eta_{ab} E_n^b \quad (18)$$

holds, where Eq. (16) was taken into account. This freedom to choose a convenient matrix m will be used below. It is an essential ingredient of our argument.

In terms of (17) the Nambu–Goto volume form is rewritten as

$$d^{p+1} \xi \sqrt{-\det(g_{mn})} = d^{p+1} \xi \det(E_m^a) = \frac{1}{(p+1)!} \epsilon_{a_0 \dots a_p} E^{a_0} \wedge \dots \wedge E^{a_p}. \quad (19)$$

The action (3) includes the auxiliary antisymmetric tensor superfield F_{ab} , and the $(p-1)$ -form Lagrange multiplier Q_{p+1} in addition to the world volume gauge field A_m . The Lagrange multiplier Q_{p+1} produces the equation

$$\mathcal{F} \equiv dA = \frac{1}{2} E^a \wedge E^b F_{ba} \quad (20)$$

which can be solved algebraically with respect to the auxiliary field F_{ab} expressing it in terms of the field strength \mathcal{F}_{mn} (2). On the other hand, the variation of the action with respect to the auxiliary field F_{ab} yields

$$Q_{p-1} \wedge E^b \wedge E^a = \frac{\sqrt{\det(\eta + F)}}{(p+1)!} E^{a_0} \wedge \dots \wedge E^{a_p} \epsilon_{a_0 \dots a_p} (\eta + F)^{-1 [ba]} \quad (21)$$

whose solution for Q_{p-1} is

$$Q_{p+1} = \frac{\sqrt{\det(\eta + F)}}{2(p-1)!} E^{a_1} \wedge \dots \wedge E^{a_{p-1}} \epsilon_{a_1 \dots a_{p-1} bc} (\eta + F)^{-1 bc}. \quad (22)$$

Thus Q_{p-1} does not contain propagating degrees of freedom.

The dynamical equations appear in the moving frame formulation as follows. One varies the action with respect to the embedding functions $X^{\underline{m}}$ and the gauge fields A_m

$$d \left(\left(\frac{1}{p!} E^{a_1} \wedge \dots \wedge E^{a_p} \epsilon_{a_1 \dots a_p a} \sqrt{-\det(\eta_{ab} + F_{ab})} - Q_{p-1} \wedge E^b F_{ab} \right) u_{\underline{m}}^a \right) = 0, \quad (23)$$

$$dQ_{p-1} = 0 \quad (24)$$

and uses the algebraic equations (16), (20), (22) to exclude auxiliary variables from Eqs. (23), (24). Replacing the induced vielbein by the induced metric (18) one then reproduces the equations following from (1). In this way the classical equivalence of the moving frame formulation of the D-p-branes (3) with the standard Born–Infeld–like one (1) can be proved.

To see this equivalence at the level of the action functionals, the algebraic equation is used to remove the auxiliary field from the functional (3). The second term (5) vanishes as a result of Eq.

(20). Henceforth the auxiliary field F_{ab} has to be replaced by the field strength $\mathcal{F}_{ab} = \mathcal{F}_{mn}\mathcal{E}_a^m\mathcal{E}_b^n$ ($\mathcal{E}_a^m E_n^a = \delta_n^m$) in the first term (4). The square root multiplier in the first term of the functional (3) may be written as $\frac{\sqrt{-\det(g_{mn} + \mathcal{F}_{mn})}}{\sqrt{-\det(g_{mn})}}$, where g_{mn} is the induced metric (18). Then, using the consequence (19) of the algebraic equation (16), one gets the standard DBI-like action (1) (see [9] for more details as well as for the supersymmetric generalization).

3 The adequate world volume vielbein.

The moving frame action for strings and for *type I* p-branes [20, 14] can be written as [23]³

$$S_I = \int_{\mathcal{M}^{p+1}} \frac{1}{(p+1)!} E^{a_0} \wedge \dots \wedge E^{a_p} \epsilon_{a_0 \dots a_p}. \quad (25)$$

However, its original form was of the first order with respect to the X variable [20, 14]

$$S'_I = \int_{\mathcal{M}^{p+1}} \left(\frac{1}{p!} E^{a_0} \wedge e^{a_1} \wedge \dots \wedge e^{a_p} \epsilon_{a_0 \dots a_p} - \frac{1}{(p+1)!} e^{a_0} \wedge e^{a_1} \wedge \dots \wedge e^{a_p} \epsilon_{a_0 \dots a_p} \right) \quad (26)$$

where $e^a = d\xi^m e_m^a(\xi)$ is the (auxiliary) world volume vielbein 1-form field.

The action functional (26) in addition to the equation (16) produces Eq. (17) with unit matrix m

$$E^a \equiv dX^m u_{\underline{m}}^a = e^a. \quad (27)$$

Such an identification of the intrinsic world volume vielbein with the induced one is indeed natural for *type I* extended objects (superbranes). For the supersymmetric case it results in the standard expression (constraint) for the dimension 1 bosonic world volume torsion superform [13]

$$T_{\alpha q \beta p}^a \propto \gamma_{\alpha\beta}^a C_{qp}, \quad (28)$$

and the fermionic equations acquire the standard form with vanishing gamma trace of a spin 3/2 superfield

$$\gamma_{\beta\alpha}^a \psi^{a\alpha q} = 0. \quad (29)$$

The linearized approximation of (29) is given essentially by the vector derivative of the target space Grassmann coordinate superfield Θ^μ

$$\psi_a^{\alpha q} \approx \partial_a \Theta^\mu v_{\underline{\mu}}^{\alpha q}. \quad (30)$$

In Eqs. (28), (29), (30) α and q are $SO(1, p)$ and $SO(D - p - 1)$ spinor indices carried by a world volume Grassmann covariant derivative $D_{\alpha q}$ and by a world volume Grassmann vielbein form $e^{\alpha q}$. $\gamma_{\alpha\beta}^a$ are $SO(1, p)$ sigma matrices and C_{qp} is second rank invariant spin-tensor of the $SO(D - p - 1)$ group. $v_{\underline{\mu}}^{\alpha q}$ denotes the $\frac{SO(1, D-1)}{SO(1, p) \times SO(D-p-1)}$ spinor Lorentz harmonic variable (see [14] and refs. therein).

To reach the same standard structure for the case of $D = 10$ *type II* super-Dp-branes [3, 9] (and the M-theory 5-brane [22]), the invertible matrix m (17) must be chosen to be depend on the

³Eq. (25) superficially coincides with (19), however it has the nontrivial property to produce Eq. (16) which is necessary for the identification with Nambu-Goto action (19).

field strength \mathcal{F} of the world volume vector field (or of the selfdual worldvolume antisymmetric tensor field of the five-brane [22]) [9].

Thus for the case of a *type IIB* 3-brane which we will consider below, an adequate choice of the matrix m is given by [9]

$$m_a{}^b = \delta_a{}^b + \frac{1}{z(F)\bar{z}(F)} \text{Tr}(\tilde{\sigma}_a F \sigma^b \bar{F}). \quad (31)$$

In the trace of (31) F , \bar{F} are spinor representations for the selfdual and the anti-selfdual part of the tensor F_{ab}

$$F_{\alpha\beta} = F_{\beta\alpha} = \frac{i}{4} F^{ab} (\sigma_a \tilde{\sigma}_b)_{\alpha\beta}, \quad \bar{F}_{\dot{\alpha}\dot{\beta}} = \bar{F}_{\dot{\beta}\dot{\alpha}} = -\frac{i}{4} F^{ab} (\tilde{\sigma}_a \sigma_b)_{\dot{\alpha}\dot{\beta}}, \quad (32)$$

$$F_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} \equiv F_a{}^b \sigma_{\alpha\dot{\alpha}}^a \tilde{\sigma}_b^{\dot{\beta}\beta} = 2\delta_{\alpha}^{\beta} \bar{F}_{\dot{\alpha}}^{\dot{\beta}} + 2\delta_{\dot{\alpha}}^{\dot{\beta}} F_{\alpha}^{\beta}, \quad *F_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} = 2\delta_{\alpha}^{\beta} \bar{F}_{\dot{\alpha}}^{\dot{\beta}} - 2\delta_{\dot{\alpha}}^{\dot{\beta}} F_{\alpha}^{\beta}, \quad (33)$$

and the scalar factors z and its complex conjugate \bar{z} (called $\frac{1}{2b_{\pm}}$ in [9]) are expressed through the F_{ab} tensor as

$$z = \frac{1}{2} \left(1 + \frac{i}{8} \epsilon^{abcd} F_{ab} F_{cd} + \sqrt{-\det(\eta + F)} \right), \quad (34)$$

The action for the D3-brane can be represented as

$$S'_{D3} = \int_{\mathcal{M}^{1+3}} (\mathcal{L}_4^{0'} + \mathcal{L}_4^1), \quad (35)$$

where

$$\mathcal{L}_4^{0'} = \det(m) \sqrt{-\det(\eta_{ab} + F_{ab})} \left(\frac{1}{3 \cdot 3!} E^{a'} m^{-1}{}_{a'}{}^a \wedge e^b \wedge e^c \wedge e^d \epsilon_{abcd} - \frac{1}{3 \cdot 4!} e^a \wedge e^b \wedge e^c \wedge e^d \epsilon_{abcd} \right) \quad (36)$$

can be obtained from (26) if one replaces e^a by $e^b m_b{}^a$, includes an overall multiplier $\sqrt{-\det(\eta + F)}$ (see (4)) and uses the identities

$$\epsilon_{abcd} m_a{}^a{} m_b{}^b{} m_c{}^c{} m_d{}^d{} = \epsilon_{a'b'c'd'} \det(m), \quad \epsilon_{abcd} m_b{}^b{} m_c{}^c{} m_d{}^d{} = \det(m) m^{-1}{}_{a'}{}^a{} \epsilon_{a'b'c'd'},$$

The second term in the functional (35)

$$\mathcal{L}_4^1 = Q_2 \wedge [e^{-\frac{\phi}{2}} (dA - B_{(2)}) - \frac{1}{2} E^b \wedge E^a F_{ab}], \quad (37)$$

corresponds to (5), but (in contrast to (5)) with the dependence on dilaton $\phi = \phi(X(\xi))$ and NS-NS 2-form background field $B_2 = \frac{1}{2} dX^m \wedge dX^n B_{nm}(X)$ restored.

However, the drawback of such an action is the complicated form of the dependence on the antisymmetric tensor field F_{ab} . This indicates that the latter is not an appropriate auxiliary field for the problem under consideration.

4 Adequate auxiliary variables and a new first order form of the D_3 -brane action

The superfield (embedding) equations [14] for type *II* super- D_3 -brane [3, 9] include the symmetric spin tensor (super)fields $h_{\alpha\beta}$, $\bar{h}_{\dot{\alpha}\dot{\beta}}$. They are expressed in terms of anti-selfdual and selfdual components $F_{\alpha\beta}$ and $\bar{F}_{\dot{\alpha}\dot{\beta}}$ of the antisymmetric tensor F_{ab} by

$$h_{\alpha\beta} = \frac{1}{z(F)} F^{\alpha\beta}, \quad \bar{h}_{\dot{\alpha}\dot{\beta}} = \frac{1}{\bar{z}(F)} \bar{F}_{\dot{\alpha}\dot{\beta}}, \quad (38)$$

where the functions z, \bar{z} are defined in Eq. (34). The expression (31) for the matrix m simplifies when written in the terms of these spin tensors

$$m_a^b = \delta_a^b + \frac{1}{2} Sp(\tilde{\sigma}_a h \sigma^b \bar{h}), \quad \Leftrightarrow \quad m_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} \equiv m_a^b \sigma_{\alpha\dot{\alpha}}^a \tilde{\sigma}_b^{\beta\dot{\beta}} = 2(\delta_{\alpha}^{\beta} \delta_{\dot{\alpha}}^{\dot{\beta}} + h_{\alpha}^{\beta} \bar{h}_{\dot{\alpha}}^{\dot{\beta}}). \quad (39)$$

This provides us with (indirect) evidence that precisely these spin tensors $h_{\alpha\beta}, \bar{h}_{\dot{\alpha}\dot{\beta}}$ are adequate auxiliary variables for the first order (super-)D3-brane action.

If this is true, then all we need to do is to express the tensor F_{ab} in terms of the spin-tensors h and \bar{h} by solving Eq. (38).

From Eqs. (34) we have

$$z + \bar{z} = 1 + \sqrt{-\det(\eta + F)}, \quad z - \bar{z} = \frac{i}{8} \epsilon^{abcd} F_{ab} F_{cd} \quad (40)$$

whereas from Eqs. (32) and (38) we obtain ⁴

$$\frac{i}{4} \epsilon^{abcd} F_{ab} F_{cd} \equiv \frac{1}{2} F_{ab}^* F^{ba} = z^2 h^2 - \bar{z}^2 \bar{h}^2, \quad \frac{1}{2} F_{ab} F^{ba} = -z^2 h^2 - \bar{z}^2 \bar{h}^2, \quad (41)$$

with the abbreviations

$$h^2 \equiv h_{\alpha\beta} h^{\alpha\beta}, \quad \bar{h}^2 \equiv \bar{h}_{\dot{\alpha}\dot{\beta}} \bar{h}^{\dot{\alpha}\dot{\beta}}. \quad (42)$$

These relations and the identity

$$-\det(\eta + F) \equiv 1 - \frac{1}{2} F_{ab} F^{ba} - \left(\frac{1}{8} \epsilon^{abcd} F_{ab} F_{cd}\right)^2$$

can be used to write the product of z with \bar{z} (34) as

$$4z\bar{z} = \left(1 + \sqrt{-\det(\eta + F)}\right)^2 + \left(\frac{1}{8} \epsilon^{abcd} F_{ab} F_{cd}\right)^2 = 2(z + \bar{z}) + z^2 h^2 + \bar{z}^2 \bar{h}^2. \quad (43)$$

From the second Eq. (40) together with (41) and replacing the first Eq. (40) by (43) we arrive at

$$z - \bar{z} = \frac{1}{2}(\bar{z}^2 \bar{h}^2 - z^2 h^2), \quad z + \bar{z} = 2z\bar{z} - \frac{1}{2}(\bar{z}^2 \bar{h}^2 + z^2 h^2)., \quad (44)$$

The sum and the difference of these equations are homogeneous in z and \bar{z} , respectively. Thus for nonvanishing z (as implied by Eq. (40)) we can extract a system of linear equations for z and \bar{z}

$$1 = \bar{z} - \frac{1}{2} h^2 z, \quad 1 = z - \frac{1}{2} \bar{h}^2 \bar{z}, \quad (45)$$

with the solution

$$z = \frac{1 + \frac{1}{2} \bar{h}^2}{1 - \frac{1}{4} h^2 \bar{h}^2}, \quad \bar{z} = \frac{1 + \frac{1}{2} h^2}{1 - \frac{1}{4} h^2 \bar{h}^2}. \quad (46)$$

Substituting (46) into (38) we obtain the expression for the selfdual and anti-selfdual parts (32) of F_{ab} and, hence, for the whole tensor F_{ab} from (33). The expression for the DBI-like square root can be obtained directly from Eqs. (46) and (40):

$$\sqrt{-\det(\eta + F)} = \frac{(1 + \frac{h^2}{2})(1 + \frac{\bar{h}^2}{2})}{1 - \frac{h^2 \bar{h}^2}{4}} \quad (47)$$

⁴An even more general identity $F_{ac}^* F^{cb} = \frac{1}{2} \delta_a^b (z^2 h^2 - \bar{z}^2 \bar{h}^2)$ can be proved using the spinor representation (33).

The inverse matrix m^{-1} and the determinant $\det(m)$ are

$$m^{-1}{}_a{}^b = \left(\delta_a{}^b - \frac{1}{2} Sp(\tilde{\sigma}_a h \sigma^b \bar{h}) \right) \frac{1}{\sqrt{\det(m)}}, \quad (48)$$

$$\det(m) = \left(1 - \frac{h^2 \bar{h}^2}{4} \right)^2. \quad (49)$$

Eq. (48) can be obtained directly from the spinor representation for the matrix m (39) while the simplest way to obtain (49) is to use a special gauge, where only two of the components of the tensor F_{ab}

$$F_{01} = f_+, \quad F_{34} = f_- \quad (50)$$

are nonvanishing (gauges of such type were used in [4, 28]). One can verify that in this gauge

$$z^2 h^2 = \frac{1}{2} (f_- + i f_+)^2, \quad \bar{z}^2 \bar{h}^2 = \frac{1}{2} (f_- - i f_+)^2$$

and (I_2 is 2×2 unit)

$$m_a{}^b = \begin{pmatrix} (1 + \frac{f_-^2 + f_+^2}{2z\bar{z}}) I_2 & 0 \\ 0 & -(1 - \frac{f_-^2 + f_+^2}{2z\bar{z}}) I_2 \end{pmatrix}$$

Eq. (49) can be easily obtained from these expressions.

Substituting (47), (48), (49), into (35), (36), (37) we arrive at

$$S'_{D_3} = \int_{\mathcal{M}^{1+3}} (\mathcal{L}_4^{0'} + \mathcal{L}_4^{1'}), \quad (51)$$

$$\begin{aligned} \mathcal{L}_4^{0'} = \frac{\left(1 + \frac{h^2}{2}\right) \left(1 + \frac{\bar{h}^2}{2}\right)}{3 \cdot 3!} & \left[E^{a'} \left(\delta_{a'}{}^a - \frac{1}{2} \tilde{\sigma}_{a'}{}^{\dot{\beta}\beta} h_{\beta}{}^{\alpha} \sigma_{\alpha\dot{\alpha}}^a \bar{h}^{\dot{\alpha}}{}_{\dot{\beta}} \right) \wedge e^a \wedge e^c \wedge e^d \epsilon_{abcd} - \right. \\ & \left. - \frac{1}{2} \left(1 - \frac{h^2 \bar{h}^2}{4} \right) e^a \wedge e^b \wedge e^c \wedge e^d \epsilon_{abcd} \right], \end{aligned} \quad (52)$$

$$\mathcal{L}_4^{1'} = Q'_2 \wedge \left[\left(1 - \frac{h^2 \bar{h}^2}{4} \right) (dA - B_{(2)}) - \frac{1}{4} E^{\alpha\dot{\alpha}} \wedge E_{\beta\dot{\beta}} \left(\delta_{\alpha}{}^{\beta} \left(1 + \frac{h^2}{2} \right) \bar{h}_{\dot{\alpha}}{}^{\dot{\beta}} + \delta_{\dot{\alpha}}{}^{\dot{\beta}} \left(1 + \frac{\bar{h}^2}{2} \right) h_{\alpha}{}^{\beta} \right) \right], \quad (53)$$

where, in addition to (42) the bispinor representation for the vielbein indices $E^{\alpha\dot{\alpha}} = E^a \tilde{\sigma}_a^{\dot{\alpha}\alpha}$ are used. We recall that $h_{\alpha\beta} = h_{\beta\alpha}$, $\bar{h}_{\dot{\alpha}\dot{\beta}} = \bar{h}_{\dot{\beta}\dot{\alpha}}$ are auxiliary symmetric spin tensor fields replacing F_{ab} .

The functional (51), (52), (53) is our result for the first order action for the *type IIB* $D3$ -brane.

5 Conclusion and discussion

In this note we present a new first order form of the action functional for the Dirichlet 3-brane. It is inspired by the superfield equations of the *type IIB* super- D_3 -brane obtained in [9] from the generalized action and possesses a polynomial structure in the auxiliary spin-tensor fields $h_{\alpha\beta}$ and $\bar{h}_{\dot{\alpha}\dot{\beta}}$ which assume the place of the initial antisymmetric tensor F_{ab} . It is remarkable that

the superfield counterparts of these objects appear in the basic superfield equations (fermionic rheotropic conditions [13]) of the super- $D3$ -brane model [22, 9].

This action can be used as a basis for the reformulation of the generalized action principle for the super- $D3$ -brane in a simpler and more transparent manner, similar to the one originally proposed for superstrings and type I superbranes in [13].

The new polynomial first order action also seems very promising for further studies of dualities following the arguments presented in [10, 12].

An interesting problem for further investigations is the generalization of our result to the case of *type II* Dp-branes with $p > 3$. Of course, then we cannot apply the spinor calculus in the manner which was most convenient for the 3-brane model. The analog of the special gauge (50) should be used instead.

An object similar to the spin tensors $h_{\alpha\beta}$, $\bar{h}_{\dot{\alpha}\dot{\beta}}$ appears in the superfield equations [22] for M-theory 5-brane [27]. In that case it is a symmetric spin-tensor field $h_{\hat{\alpha}\hat{\beta}} \equiv \gamma_{\hat{\alpha}\hat{\beta}}^{\hat{a}\hat{b}\hat{c}} h_{\hat{a}\hat{b}\hat{c}}$ ($\hat{\alpha}, \hat{\beta} = 1, \dots, 4$; $\hat{a}, \hat{b}, \hat{c} = 0, \dots, 5$) which provides a spinor representation for the $d = 6$ selfdual antisymmetric tensor $h_{\hat{a}\hat{b}\hat{c}} = \frac{1}{3!} \epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}\hat{f}} h^{\hat{d}\hat{e}\hat{f}}$.

On the other hand, the original action for M-theory super-5-brane [30, 31] as well as the moving frame action described shortly in [23] contains the auxiliary scalar field [29] and a field strength of the second rank antisymmetric world volume tensor field. This field satisfies a non-linear generalization of the selfduality conditions [28] on the mass shell (in fact this selfduality only appears after gauge fixing of a special symmetry). As it was proved in [32, 33], the component equations following from the action [30, 31] coincide with the ones having been obtained from the superfield embedding equations [22]. In this way the (ordinary) selfdual tensor field h_{abc} was expressed in terms of the auxiliary scalar and the field strength of the world volume 2-form gauge field.

However, to complete the proof of the equivalence of the superfield approach [22] (based on the embedding equations without reference to any action) and the component approach based on the action [30, 31, 23], it is necessary to lift the moving frame reformulation [23] of the super-5-brane action [30, 31] to a generalized action which should produce the superfield equations. Such a program was completed for the superstring, for *type I* super-p-branes [13] and *type II* Dirichlet superbranes [9]. However, for the case of the 5-brane its realization has encountered a problem [23] related to the specific way the auxiliary scalar field is present in the action (see [34] for more details).

One of the possibilities to overcome such a difficulty consists in searching for another form of the super-5-brane action involving a spin-tensor representation $h_{\hat{\alpha}\hat{\beta}}$ of the (linearly) selfdual tensor field $h_{\hat{a}\hat{b}\hat{c}}$ instead of a specific combination of the auxiliary scalar field and the field strength of the world volume two form. Our present study demonstrates that a similar program at least can be realized for the Dirichlet 3-brane. In this sense this note can be regarded as a preliminary study of the possibility to find a reformulation of the 5-brane action, although we are well aware of additional problems which are bound to appear due to the more complicated structure of the 5-brane theory.

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